

# Antenna Arraying Performance for Deep Space Telecommunications Systems

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*Antenna arraying will be a crucial Deep Space Network technique in maximizing the science return of planetary and comet encounters in the 1980's. This article develops the equations which describe the total figure of merit for a multiple system of arrayed antennas. An example is given for three Canberra DSN antennas and the Parkes 64-m antenna to be arrayed for the Voyager 2 Uranus flyby.*

## I. Introduction

As the Deep Space Network prepares for the decade of the 1980's, the science return for critical spacecraft encounters such as Voyager 2 at Uranus and Neptune and Giotto at Halley's Comet is expected to be significantly enhanced through the use of multiple antenna arraying. Antenna arraying concepts were developed in the 1960's for radio astronomy and were successfully demonstrated for space telecommunications with Pioneer 8 (Ref. 1) and during the Mariner 10 Mercury encounter in 1974 (Refs. 2 and 3) and the Voyager 2 Saturn encounter in 1981 (Ref. 4).

The figure of merit of a single aperture receiving system is given by (Ref. 5)<sup>1</sup>

$$M = G_R / T_{op} \text{ , } K^{-1} \quad (1)$$

where

$G_R$  = receiving antenna gain, ratio

$T_{op}$  = system noise temperature, K

A profile of the figure of merit of the Deep Space Network (DSN) receiving systems from 1960 to 1982 is shown in Fig. 1.

The figure of merit required for a receiving system to support a given communication link is (Ref. 6)

$$M \geq 4\pi kRLD^2 (E_b/N_0)_T / P_T A_T \quad (2)$$

where

$D$  = distance between the transmitting and receiving antennas, m

$R$  = data bit rate, bps

$A_T$  = effective area of transmitting antenna, m<sup>2</sup>

<sup>1</sup>For convenience,  $M$  is usually used in computations as shown with units  $K^{-1}$  and in discussion with units of dB where  $M \text{ (dB)} = 10 \log M$ ; it is understood that this is relative to a reference figure of merit of  $1 K^{-1}$ .

$L$  = total link losses (includes polarization loss, pointing loss, atmospheric loss, demodulation loss, etc.), ratio

$(E_b/N_0)_T$  = threshold value (Ref. 5) of  $(E_b/N_0)$  for a given bit error rate (BER), ratio

$P_T$  = transmitter power, W

$k$  = Boltzmann's constant,  $1.3806 \times 10^{-23}$ , J/K

For example, Voyager 2 at Saturn had the parameter values shown in Table 1 for transmitting imaging data. The threshold figure of merit  $M_T$  from Eq. (2) is calculated to be 55 dB. This results in a communications link margin of 3 dB (Fig. 1). This link margin is based on mean parameter values; adequate planned margin is required to allow for variations in the actual parameter values, particularly the change of  $T_{op}$  as a function of weather.

To meet future requirements for Voyager 2 communications at Uranus and Neptune, it will be necessary to improve the figure of merit with a system of arrayed apertures. This report reviews the equations which describe the figure of merit for a multiple array of antennas.

## II. Antenna Arraying

The figure of merit required for the receiving system can be obtained with an array of antennas each with a separate figure of merit ( $M_i = G_i/T_i$ ). Refs. 1, 2, and 3 developed the array performance analysis and optimum combiner strategy in terms of the receiver-detected SNR. The array figure of merit could be deduced from this analysis or derived directly. For completeness, an analysis for  $M$  in terms of  $M_i$  follows.

Assume an array of antennas (Fig. 2) where the voltage outputs of the individual channels are weighted by  $\beta_i$  and optimally combined<sup>2</sup> to maximize the output signal-to-noise ratio (SNR). Using subscript 1 for the "reference" channel, summing the coherent signal voltages ( $\sqrt{G_i g_i} \beta_i$ ) and the assumed incoherent<sup>3</sup> noise powers ( $T_i g_i \beta_i^2$ ),

$$\frac{S(\beta_i)}{S_1} = \frac{\left( \sum_{i=1}^n \sqrt{G_i g_i} \beta_i \right)^2}{G_1 g_1 \beta_1^2} \quad (3)$$

and

$$\frac{N(\beta_i)}{N_1} = \frac{\sum_{i=1}^n T_i g_i \beta_i^2}{T_1 g_1 \beta_1^2} \quad (4)$$

where

$S(\beta_i)/S_1$  = arrayed received signal power as a function of  $\beta_i$ , relative to the reference channel, ratio

$N(\beta_i)/N_1$  = arrayed noise power output as a function of  $\beta_i$ , relative to the reference channel, ratio

$G_i$  = gain of  $i$ th receiving antenna, ratio

$g_i$  = gain of  $i$ th receiver, ratio

$T_i$  = operating system noise temperature of  $i$ th antenna, K

$\beta_i$  = voltage weighting function of  $i$ th channel, ratio

Then the signal-to-noise ratio as a function of  $\beta_i$  is given by

$$SNR(\beta_i) = \frac{(SNR)_1}{M_1} \frac{\left( \sum_{i=1}^n \sqrt{G_i g_i} \beta_i \right)^2}{\sum_{i=1}^n T_i g_i \beta_i^2} \quad (5)$$

The value for  $\beta_i$  to maximize the SNR can now be obtained.

Differentiating with respect to  $\beta_i$  and setting the result equal to zero,<sup>4</sup>

<sup>4</sup>The power weighting is given by

$$(\beta_i/\beta_1)^2 = \left( \frac{T_1}{T_i} \right)^2 \frac{G_1 g_1}{G_i g_i} = \frac{T_1 g_1}{T_i g_i} \frac{(SNR)_i}{(SNR)_1}$$

This can be obtained (Ref. 7) in an operational system by first setting each channel receiver gain for equal noise level ( $g_i = g_1 T_1/T_i$ ) and then weighting each channel by an additional factor  $(SNR)_i/(SNR)_1$ . Techniques are available (Ref. 8) for monitoring  $(SNR)_i$ .

<sup>2</sup>This analysis assumes perfect combining and does not allow for unequal atmospheric losses, combiner losses, differential time delays, and other nonideal effects.

<sup>3</sup>The assumption of incoherent noise cannot be made for that portion of received noise that is due to a hot body in view of all elements of the array.

$$(\beta_i/\beta_1) = (T_1/T_i) \sqrt{G_{E1}/G_1 g_i} \quad (6)$$

and

$$SNR = \frac{(SNR)_1}{M_1} \sum_{i=1}^n M_i \quad (7)$$

Using  $SNR/(SNR)_1 = M/M_1$  results in

$$M = \sum_{i=1}^n M_i \quad (8)$$

Expanding Eq. (7) and using  $(SNR)_i/(SNR)_1 = M_i/M_1$ ,

$$SNR = \sum_{i=1}^n (SNR)_i \quad (9)$$

In terms of antenna efficiency,  $\epsilon$  and physical diameter  $D$  [using  $G_i = \epsilon_i(\pi D_i/\lambda)^2$ ],

$$M = (\pi/\lambda)^2 \sum_{i=1}^n \epsilon_i D_i^2 / T_i \quad (10)$$

Relative to the highest performance antenna with figure of merit  $M_1$ , using<sup>5</sup>  $\Delta M_i(\text{dB}) = M_1(\text{dB}) - M_i(\text{dB})$ ,

$$\Delta M(\text{dB}) = 10 \log \sum_{i=1}^n 10^{-\Delta M_i(\text{dB})/10} \quad (11)$$

<sup>5</sup> $M(\text{dB}) = 10 \log M$ .

The following section presents some examples.

### III. Discussion

Consider an idealized array of one 64-m antenna and  $N$  34-m antennas with equal antenna efficiencies and system noise temperatures. Figure 3 shows the improvements of  $N$  34-m antennas relative to the single 64-m antenna (upper curve) and relative to the single 64-m antenna plus  $(N - 1)$  34-m antennas (lower curve). This illustrates that adding multiple antennas to an array has a smaller percentage effect as  $N$  grows large.

A successful DSN array configuration was used at the Voyager 1 and 2 Saturn flybys using a 64-m antenna and a 34-m antenna (Table 1 and Fig. 1). This resulted in an average measured increase in signal-noise ratio of about 0.6 dB relative to the 64-m antenna only (Ref. 4). Assuming 50% efficiency for both antennas and no array losses, the potential improvement was about 1.1 dB (Fig. 1). Future refinements of the array techniques should reduce the implied  $\approx 0.5$  dB average array loss.

Finally, consider the optimum improvement at 8.42 GHz of the following array relative to the DSS 43 antenna: DSS 43 (64-m), DSS 42 (34-m,  $G/T = -6.0$  dB relative to DSS 43), DSS 45 (34-m,  $G/T = -4.5$  dB relative to DSS 43), and Parkes (64-m,  $G/T = -1.1$  dB relative to DSS 43). Using Eq. (11), the estimated improvement in potential figure of merit is

$$\Delta M \approx 10 \log (1 + 0.25 + 0.35 + 0.78) \text{ dB}$$

$$\approx 3.8 \text{ dB}$$

This is the array configuration and potential performance improvement presently planned for the Voyager 2 1986 Uranus encounter (Refs. 9 and 10).

## References

1. Urech, J. M., "Telemetry Improvement Proposal for the 85-ft Antenna Network," *SPS 37-63 Vol. II*, p. 116, Jet Propulsion Laboratory, Pasadena, Calif., May 31, 1970.
2. Wilck, H., "A Signal Combiner for Antenna Arraying," *DSN Progress Report 42-25*, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1975.
3. Winkelstein, R. A., "Analysis of the Signal Combiner for Multiple Antenna Arraying," *DSN Progress Report 42-26*, Jet Propulsion Laboratory, Pasadena, Calif., Apr. 15, 1975.
4. Bartok, C. D., "Performance of the Real-Time Array Signal Combiner During the Voyager Mission," *TDA Progress Report 42-63*, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1981.
5. Wait, D. F., "Satellite Earth Terminal G/T Measurements," *Microwave Journal*, Vol. 20, No. 4, Apr. 1977.
6. Stelzried, C. T., and Noreen, G. K., "The System View," *The Deep Space Network — A Radio Communications Instrument for Deep Space Exploration*, N. A. Renzetti and C. T. Stelzried, eds., Publication 82-104, Jet Propulsion Laboratory, Pasadena, Calif., February 1983.
7. Clauss, R. C., private communication, Dec. 1982.
8. Howard, L. D., "Split-Symbol Correlator Signal-to-Noise Ratio Detector," *New Technology Report*, JPL/NASA No. 5116/15653, March 31, 1981.
9. Brown, D. W., private communication, Dec. 1982.
10. "Deep Space Network/Flight Project Interface Design Book," Vol. 2: Proposed DSN Capability, 810-5, Rev. D, TLM-10, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 1982.

**Table 1. Tabulated downlink parameters for Voyager 2 spacecraft, Saturn (August 1981) flyby, X-band (8.42 GHz)**

Parameter	Value
Transmitter power $P_T$ , W	21.3
Spacecraft antenna effective area $A_T$ , m <sup>2</sup>	5.4
Distance $D$ , m	$1.557 \times 10^{12}$
Total link loss $L$ , ratio	1.1
Data rate $R$ , bps	$4.48 \times 10^4$
Threshold signal-to-noise ratio for $5 \times 10^{-3}$ BER $(E_b/N_0)_T$ , ratio	1.8
Threshold figure of merit, calculated from Eq. (2) $M_T$ , dB	55

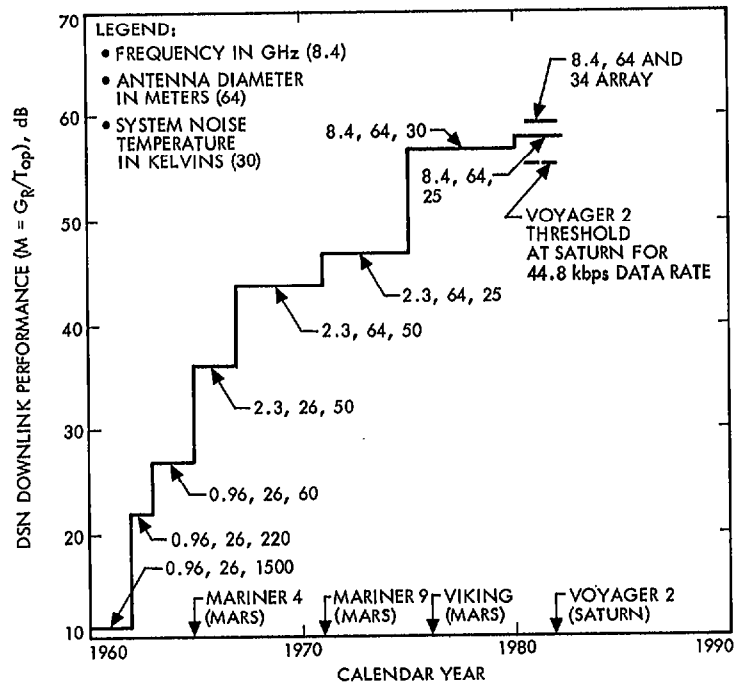


Fig. 1. Profile of the DSN downlink performance ( $M = G_R/T_{op}$ ) from 1960 to 1982

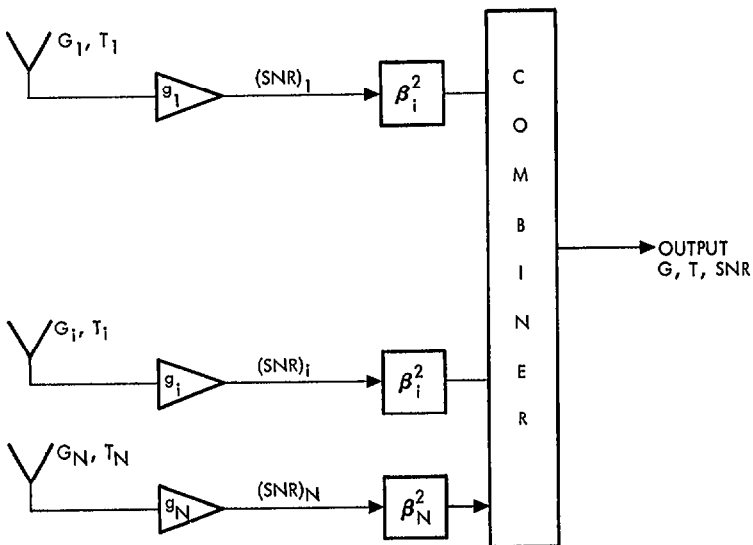


Fig. 2. Antenna array configuration

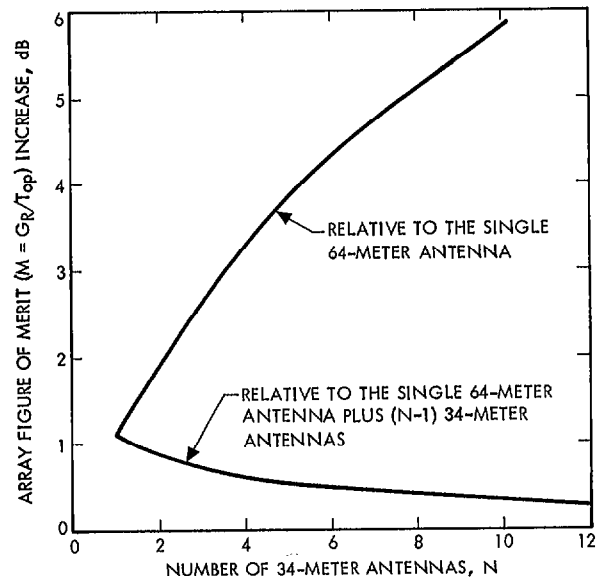


Fig. 3. Ideal figure of merit increase of an antenna array consisting of one 64-m antenna and  $N$  34-m antennas